

## त्रिकोणमिति का परिचय - 8

**उदाहरण-1** यदि  $\tan A = \frac{4}{3}$  हैं तो कोण A के अन्य त्रिकोणमितीय अनुपात ज्ञात कीजिए।

हल:-

$$\tan A = \frac{4}{3} = \frac{\text{लम्ब}}{\text{आधार}}$$

$$\text{लम्ब} : \text{आधार} = 4 : 3$$

$$\text{लम्ब} = 4K$$

$$\text{आधार} = 3K$$

$\triangle ABC$  में,

पाइथागोरस प्रमेय से,  
 $(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$

$$\Rightarrow (4K)^2 + (3K)^2 = (AC)^2$$

$$\Rightarrow 16K^2 + 9K^2 = AC^2$$

$$\Rightarrow 25K^2 = AC^2$$

$$\Rightarrow AC = \sqrt{25K^2}$$

$$\Rightarrow \boxed{AC = 5K}$$

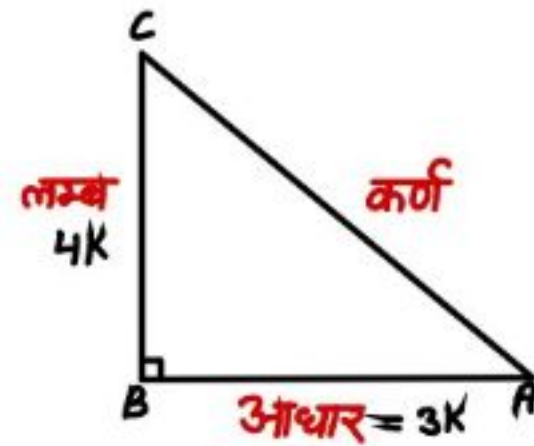
$$\sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{4K}{5K} = \frac{4}{5}$$

$$\cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{3K}{5K} = \frac{3}{5}$$

$$\cot A = \frac{\text{आधार}}{\text{लम्ब}} = \frac{3K}{4K} = \frac{3}{4}$$

$$\sec A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{5K}{3K} = \frac{5}{3}$$

$$\operatorname{cosec} A = \frac{\text{कर्ण}}{\text{लम्ब}} = \frac{5K}{4K} = \frac{5}{4}$$



**उदाहरण-2** यदि  $\angle B$  और  $\angle Q$  ऐसे समकोण हो जिससे कि  $\sin B = \sin Q$ , तो सिद्ध कीजिए कि  $\angle B = \angle Q$

हल:-

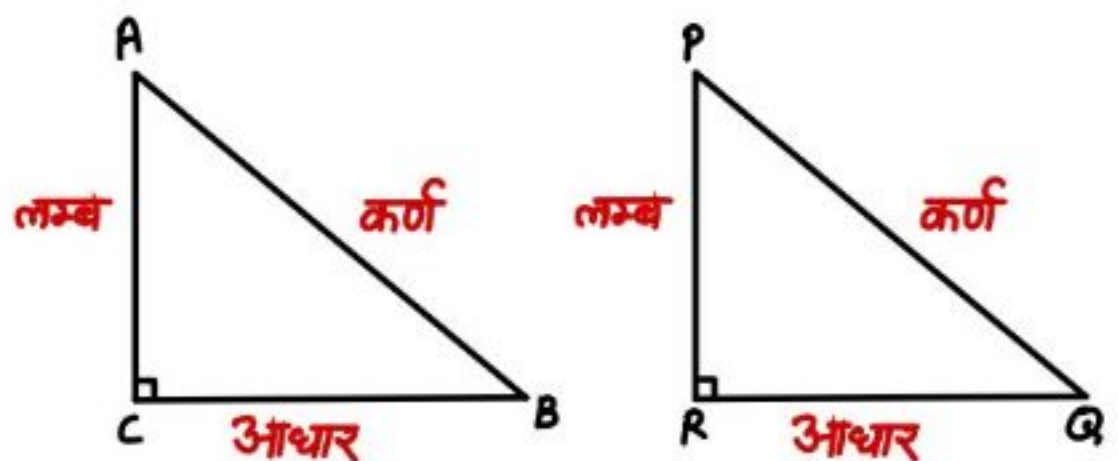
दिया है:-

$$\sin B = \sin Q$$

$$\sin B = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{AC}{AB}$$

$$\sin Q = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{PR}{PQ}$$

$$\frac{AC}{AB} = \frac{PR}{PQ} \Rightarrow \boxed{\frac{AC}{PR} = \frac{AB}{PQ}} \quad - (1)$$



$$\Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = K \text{ (मान)}$$

$$\Rightarrow \frac{AC}{PR} = K \Rightarrow AC = K(PR) - (2)$$

$$\Rightarrow \frac{AB}{PQ} = K \Rightarrow AB = K(PQ) - (3)$$

$\Delta ABC$  व  $\Delta PQR$  में,

पाइथागोरस प्रमेय से,

$$AC^2 + BC^2 = AB^2, \quad PR^2 + RQ^2 = PQ^2 - (4)$$

$$\Rightarrow [K(PR)]^2 + BC^2 = [K(PQ)]^2$$

$$\Rightarrow K^2(PR)^2 + BC^2 = K^2(PQ)^2$$

$$\Rightarrow BC^2 = K^2(PQ)^2 - K^2(PR)^2$$

$$\Rightarrow BC^2 = K^2(PQ^2 - PR^2)$$

$$\Rightarrow BC = \sqrt{K^2(PQ^2 - PR^2)}$$

$$\Rightarrow BC = \sqrt{K^2(PR^2 + RQ^2 - PR^2)}$$

$$\Rightarrow BC = K(RQ)$$

$$\Rightarrow \frac{BC}{QR} = \frac{K(RQ)}{(RQ)}$$

$$\Rightarrow \frac{BC}{QR} = \frac{K(RQ)}{(RQ)}$$

$$\Rightarrow \frac{BC}{QR} = K$$

$$\Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR} \quad \text{तीनों भुजाओं के अनुपात बराबर हैं।}$$

अतः भुजा-भुजा-भुजा समरूपता कसौटी से दोनों त्रिभुज समरूप हैं।  
इसलिए संगत कोण बराबर होंगे।

$$\Delta ABC \sim \Delta PQR$$

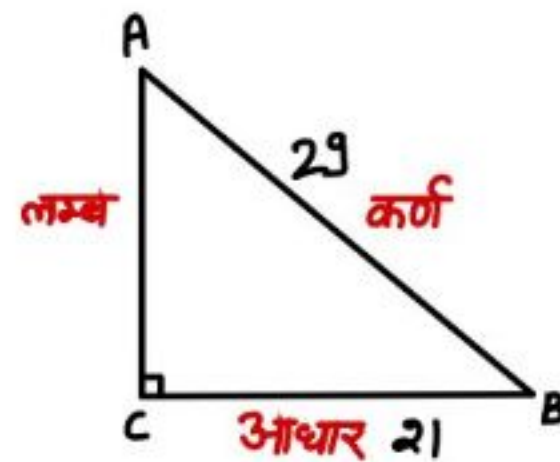
**उदाहरण-3**  $\Delta ACB$  लीजिए जिसका कोण  $C$  समकोण है जिसमें  $AB = 29$  इकाई,  $BC = 21$  इकाई और  $\angle ABC = \theta$  हैं तो निम्नलिखित के मान ज्ञात कीजिए:-

हल :-

$\Delta ACB$  में,

पाइथागोरस प्रमेय से,  
(लम्ब)<sup>2</sup> + (आधार)<sup>2</sup> = (कर्ण)<sup>2</sup>

$$\begin{aligned}
 \Rightarrow AC^2 + (21)^2 &= (29)^2 \\
 \Rightarrow AC^2 + 441 &= 841 \\
 \Rightarrow AC^2 &= 841 - 441 \\
 \Rightarrow AC^2 &= 400 \\
 \Rightarrow AC &= \sqrt{400} \\
 \Rightarrow AC &= 20 \text{ इकाई}
 \end{aligned}$$



$$\Rightarrow \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{20}{29}$$

$$\Rightarrow \cos \theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{21}{29}$$

$$(i) \cos^2 \theta + \sin^2 \theta$$

$$= \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2$$

$$= \frac{441}{841} + \frac{400}{841}$$

$$= \frac{441 + 400}{841} = \frac{841}{841} = 1$$

$$(ii) \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2$$

$$= \frac{441}{841} - \frac{400}{841}$$

$$= \frac{441 - 400}{841} = \frac{41}{841}$$

**उदाहरण-4** एक समकोण त्रिभुज ABC में, जिसका कोण B समकोण है, यदि  $\tan A = 1$  तो सत्यापित कीजिए कि  $2 \sin A \cos A = 1$

हल:-

$$\tan A = 1 = \frac{\text{लम्ब}}{\text{आधार}}$$

$$\text{लम्ब} : \text{आधार} = 1 : 1$$

$$\text{लम्ब} = 1K = K$$

$$\text{आधार} = 1K = K$$

$\Delta ABC$  में,

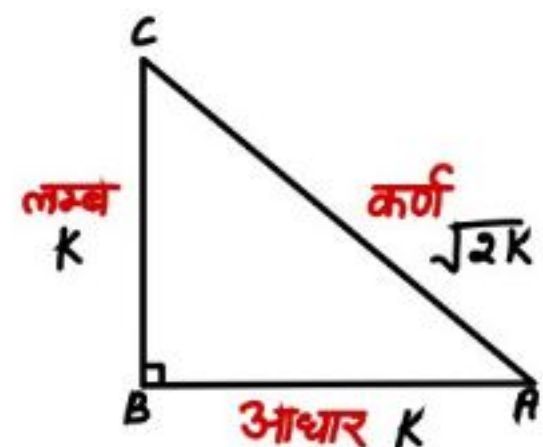
पाइथागोरस प्रमेय से,

$$(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$$

$$\Rightarrow K^2 + K^2 = AC^2$$

$$2K^2 = AC^2$$

$$AC = \sqrt{2K^2} \Rightarrow AC = \sqrt{2}K$$





$$\Rightarrow \sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{K}{\sqrt{3}K} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{K}{\sqrt{3}K} = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \text{LHS} &= 2 \sin A \cos A \\ &= 2 \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \\ &= \frac{2}{3} = 1 \\ &= \text{RHS} \end{aligned}$$

**उदाहरण-5**  $\Delta OPQ$  में, जिसका कोण  $P$  समकोण है,  $OP = 7 \text{ cm}$  और  $OQ - PQ = 1 \text{ cm}$ .  $\sin \theta$  और  $\cos \theta$  के मान ज्ञात कीजिए।

हल :-

$$\begin{aligned} OQ - PQ &= 1 \\ \Rightarrow OQ &= 1 + PQ \quad \text{--- (1)} \end{aligned}$$

$\Delta OPQ$  में,

पाइथागोरस प्रमेय से,  
 $(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$

$$\begin{aligned} \Rightarrow 7^2 + (PQ)^2 &= (1 + PQ)^2 \\ \Rightarrow 49 + PQ^2 &= 1^2 + PQ^2 + 2(1)(PQ) \\ \Rightarrow 49 &= 1 + 2(PQ) \\ \Rightarrow 48 &= 2(PQ) \end{aligned}$$

$$\Rightarrow PQ = \frac{48}{2} = 24 \text{ cm}$$

$$\Rightarrow \boxed{PQ = 24 \text{ cm}}$$

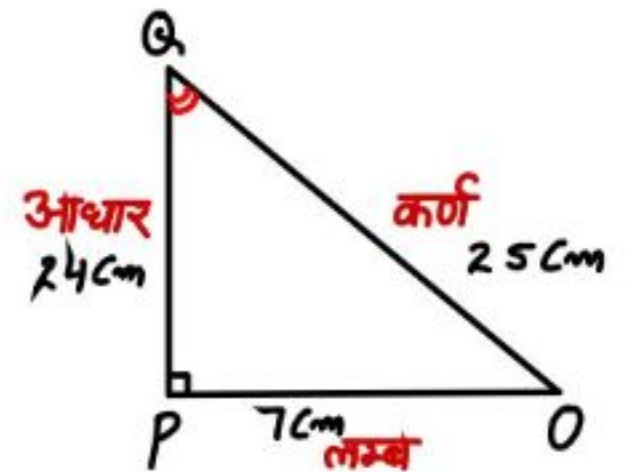
समीकरण (1) से  $\rightarrow$

$$\begin{aligned} OQ &= 1 + PQ \\ OQ &= 1 + 24 = 25 \end{aligned}$$

$$\Rightarrow \boxed{OQ = 25 \text{ cm}}$$

$$\Rightarrow \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{7}{25}$$

$$\Rightarrow \cos \theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{24}{25}$$



## प्रश्नावली 8.1

**प्रश्न-1**  $\Delta ABC$  में जिसका कोण  $B$  समकोण है,  $AB = 24\text{cm}$  और  $BC = 7\text{cm}$  है। निम्नलिखित का मान ज्ञात कीजिए।

हल:-

$\Delta ABC$  में,

$$\Rightarrow \text{पाइथागोरस प्रमेय से,}$$

$$\Rightarrow (\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$$

$$\Rightarrow AB^2 + BC^2 = AC^2$$

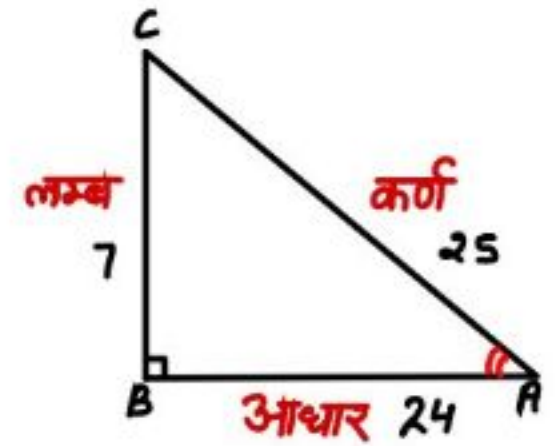
$$\Rightarrow (24)^2 + (7)^2 = AC^2$$

$$\Rightarrow 576 + 49 = AC^2$$

$$\Rightarrow 625 = AC^2$$

$$\Rightarrow AC = \sqrt{625}$$

$$\Rightarrow \boxed{AC = 25}$$



(i)  $\sin A$ ,  $\cos A$

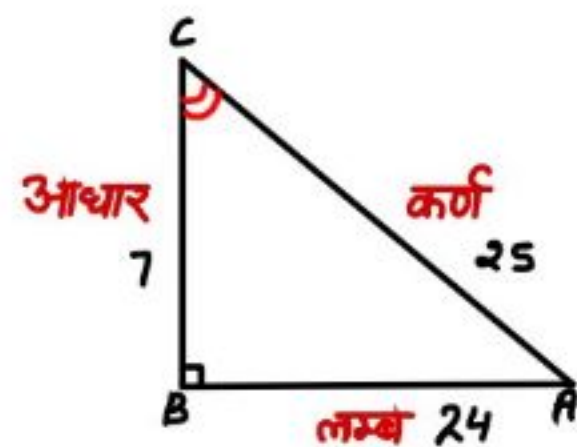
$$\sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{7}{25}$$

$$\cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{24}{25}$$

(ii)  $\sin C$ ,  $\cos C$

$$\sin C = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{24}{25}$$

$$\cos C = \frac{\text{आधार}}{\text{कर्ण}} = \frac{7}{25}$$



**प्रश्न-2** आकृति में  $\tan P - \cot R$  का मान ज्ञात कीजिए।

हल:-

$\Delta PQR$  में,

$$\text{पाइथागोरस प्रमेय से,}$$

$$(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$$

$$\Rightarrow PQ^2 + QR^2 = PR^2$$

$$\Rightarrow 12^2 + QR^2 = 13^2$$

$$\Rightarrow 144 + QR^2 = 169$$

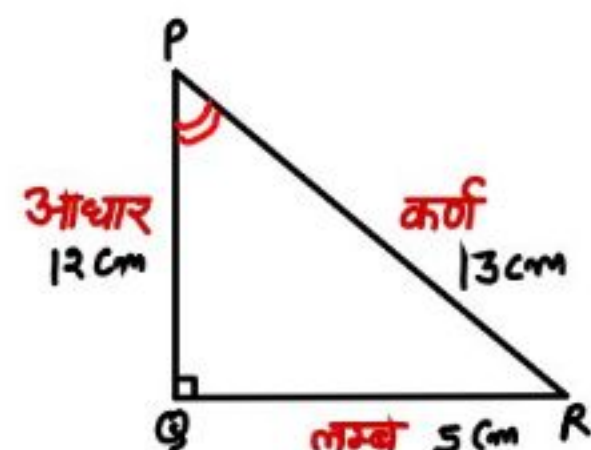
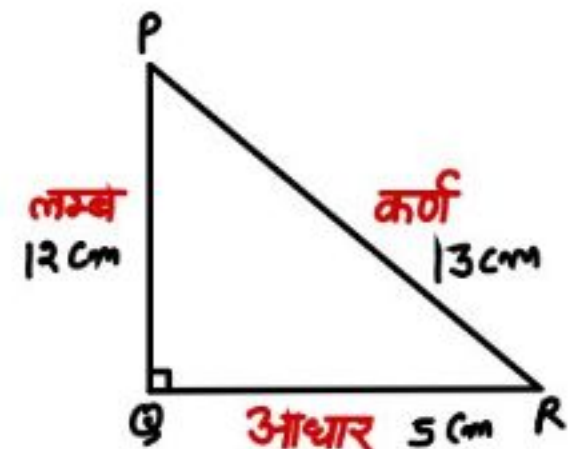
$$\Rightarrow QR^2 = 169 - 144$$

$$\Rightarrow QR^2 = 25$$

$$\Rightarrow QR = \sqrt{25}$$

$$\boxed{QR = 5\text{cm}}$$

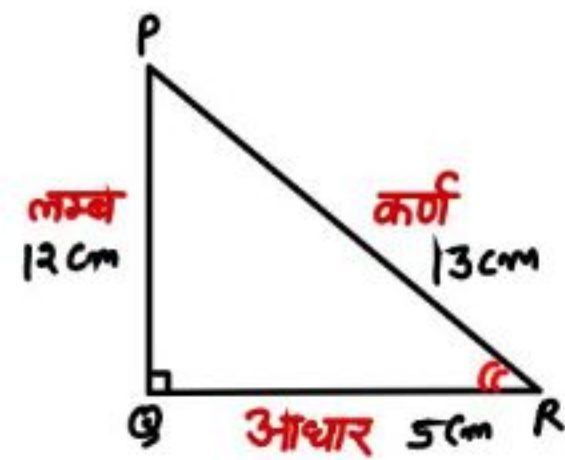
$$\Rightarrow \tan P = \frac{\text{लम्ब}}{\text{आधार}} = \frac{5}{12}$$





$$\Rightarrow \cot R = \frac{\text{आधार}}{\text{लम्ब}} = \frac{5}{12}$$

$$\begin{aligned} \text{अतः } \tan P - \cot R \\ = \frac{5}{12} - \frac{5}{12} = 0 \end{aligned}$$



**प्रश्न-3** यदि  $\sin A = \frac{3}{4}$  तो  $\cos A$  और  $\tan A$  का मान ज्ञात कीजिए।

हल:-

$$\sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{3}{4}$$

$$\text{लम्ब : कर्ण} = 3 : 4$$

माना,

$$\text{लम्ब} = 3K$$

$$\text{कर्ण} = 4K$$

$\Delta ABC$  में,

$\Rightarrow$  पाइथागोरस प्रमेय से,  
 $(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$

$$\Rightarrow AB^2 + BC^2 = AC^2$$

$$\Rightarrow AB^2 + (3K)^2 = (4K)^2$$

$$\Rightarrow AB^2 + 9K^2 = 16K^2$$

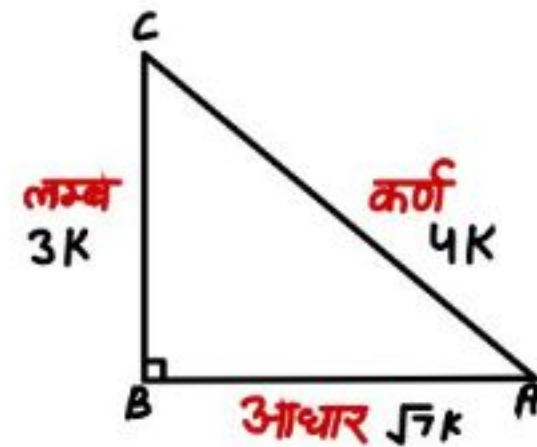
$$\Rightarrow AB^2 = 16K^2 - 9K^2$$

$$\Rightarrow AB^2 = 7K^2$$

$$\Rightarrow AB = \sqrt{7}K$$

$$\Rightarrow \cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{\sqrt{7}K}{4K} = \frac{\sqrt{7}}{4}$$

$$\Rightarrow \tan A = \frac{\text{लम्ब}}{\text{आधार}} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$$



**प्रश्न-4** यदि  $\cot A = 8$  हो तो  $\sin A$  और  $\sec A$  का मान ज्ञात कीजिए।

हल:-

$$15 \cot A = 8$$

$$\cot A = \frac{8}{15} = \frac{\text{आधार}}{\text{लम्ब}}$$

$$\text{आधार : लम्ब} = 8 : 15$$

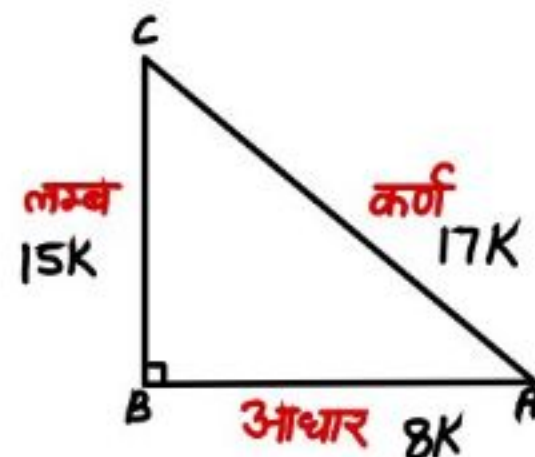
$$\text{आधार} = 8K$$

$$\text{लम्ब} = 15K$$

$\Delta ABC$  में,

$\Rightarrow$  पाइथागोरस प्रमेय से,  
 $(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$

$$\Rightarrow AB^2 + BC^2 = AC^2$$



$$\begin{aligned}\Rightarrow (8K)^2 + (15K)^2 &= AC^2 \\ \Rightarrow 64K^2 + 225K^2 &= AC^2 \\ \Rightarrow 289K^2 &= AC^2 \\ \Rightarrow AC &= \sqrt{289K^2} \\ \boxed{AC} &= \boxed{17K}\end{aligned}$$

$$\sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{15K}{17K} = \frac{15}{17}$$

$$\sec A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{17K}{8K} = \frac{17}{8}$$

**प्रश्न-5** यदि  $\sec \theta = \frac{13}{12}$  हो तो अन्य सभी त्रिकोणमितीय अनुपात परिष्कृत कीजिए।

हल:-

$$\sec A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{13}{12}$$

$$\text{कर्ण} : \text{आधार} = 13 : 12$$

$$\text{कर्ण} = 13K$$

$$\text{आधार} = 12K$$

$\Delta ABC$  में,

$\Rightarrow$  पाइथागोरस प्रमेय से,

$$(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$$

$$\Rightarrow AB^2 + AC^2 = BC^2$$

$$(12K)^2 + AC^2 = (13K)^2$$

$$144K^2 + AC^2 = 169K^2$$

$$AC^2 = 169K^2 - 144K^2$$

$$AC^2 = 25K^2$$

$$AC = \sqrt{25K^2}$$

$$\boxed{AC} = \boxed{5K}$$

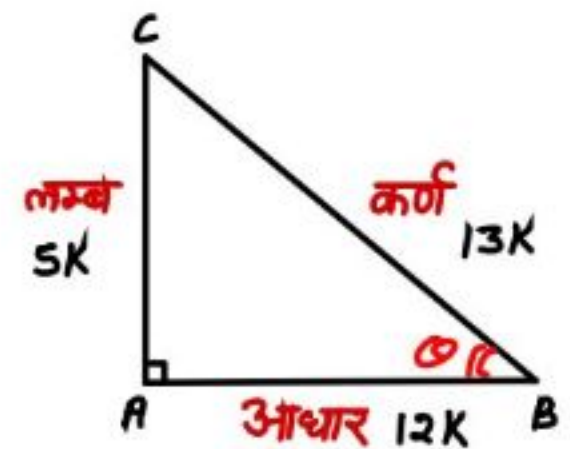
$$\Rightarrow \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{5K}{13K} = \frac{5}{13}$$

$$\Rightarrow \cos \theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{12K}{13K} = \frac{12}{13}$$

$$\Rightarrow \tan \theta = \frac{\text{लम्ब}}{\text{आधार}} = \frac{5K}{12K} = \frac{5}{12}$$

$$\Rightarrow \cot \theta = \frac{\text{आधार}}{\text{लम्ब}} = \frac{12K}{5K} = \frac{12}{5}$$

$$\sec \theta = \frac{\text{कर्ण}}{\text{आधार}} = \frac{13K}{12K} = \frac{13}{12}$$





$$\operatorname{Cosec} \theta = \frac{\text{कर्ण}}{\text{लम्ब}} = \frac{13K}{5K} = \frac{13}{5}$$

**प्रश्न-6** यदि  $\angle A$  और  $\angle B$  समकोण हो, जहाँ  $\cos A = \cos B$  तो दिखाइए कि  $\angle A = \angle B$ .  
हल:-

$$\Rightarrow \cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{AC}{AB} \quad \text{--- (1)}$$

$$\Rightarrow \cos B = \frac{\text{आधार}}{\text{कर्ण}} = \frac{BC}{AB} \quad \text{--- (2)}$$

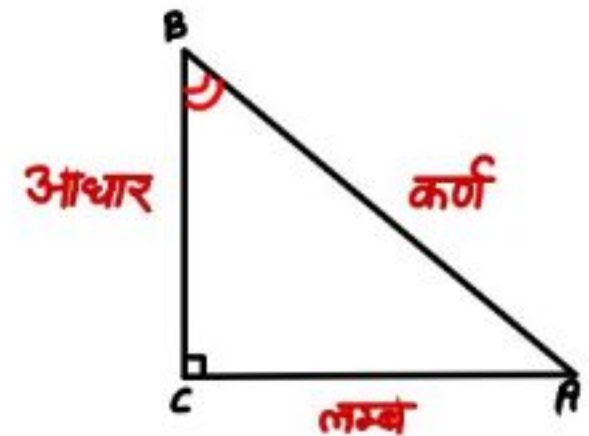
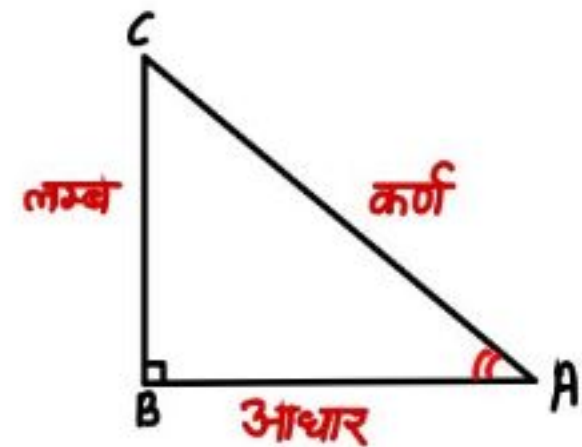
दिया है,  $\cos A = \cos B$

$$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$$

$$\Rightarrow AC = BC$$

किसी त्रिभुज में दो समान भुजाओं के सम्मुख कोण भी समान होते हैं।

अतः  $\angle B = \angle A$



**प्रश्न-7** यदि  $\cot \theta = \frac{7}{8}$  तो (i)  $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$ ,

(ii)  $\cot^2 \theta$  का मान ज्ञात कीजिए।

हल:-  $\Rightarrow \cot \theta = \frac{\text{आधार}}{\text{लम्ब}} = \frac{7}{8}$

आधार : लम्ब = 7 : 8

आधार = 7K

लम्ब = 8K

$\triangle ABC$  में,

पाइथागोरस प्रमेय से,

$$(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$$

$$AB^2 + BC^2 = AC^2$$

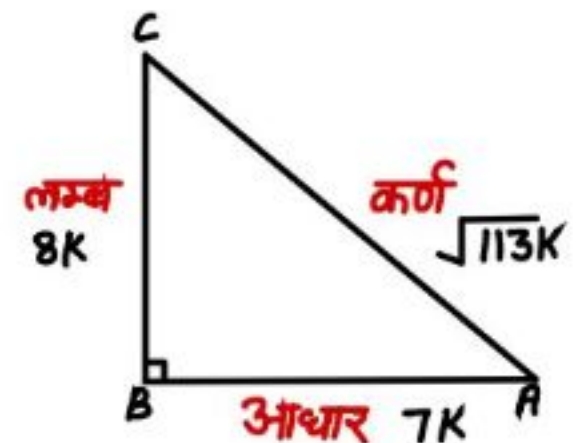
$$(7K)^2 + (8K)^2 = AC^2$$

$$49K^2 + 64K^2 = AC^2$$

$$113K^2 = AC^2$$

$$AC = \sqrt{113K^2}$$

$$= \sqrt{113} K$$





$$\Rightarrow \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{8K}{\sqrt{113}K} = \frac{8}{\sqrt{113}}$$

$$\Rightarrow \cos \theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{7K}{\sqrt{113}K} = \frac{7}{\sqrt{113}}$$

$$\Rightarrow \cot \theta = \frac{\text{आधार}}{\text{लम्ब}} = \frac{7K}{8K} = \frac{7}{8}$$

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{1^2 - (\sin \theta)^2}{1^2 - (\cos \theta)^2}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\frac{\left(1 - \frac{64}{113}\right)}{\left(1 - \frac{49}{113}\right)} = \frac{\left(\frac{113-64}{113}\right)}{\left(\frac{113-49}{113}\right)}$$

$$= \frac{\left(\frac{49}{113}\right)}{\left(\frac{64}{113}\right)} = \frac{49}{113} \times \frac{113}{64}$$

$$= \boxed{\frac{49}{64}}$$

$$(ii) \cot^2 \theta$$

$$\Rightarrow \cot \theta = \frac{\text{आधार}}{\text{लम्ब}} = \frac{7K}{8K} = \frac{7}{8}$$

$$\cot^2 \theta = \left(\frac{7}{8}\right)^2 = \boxed{\frac{49}{64}}$$

**प्रश्न-8** यदि  $3 \cot A = 4$  तो जाँच कीजिए कि  $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$  है या नहीं।

हल:-

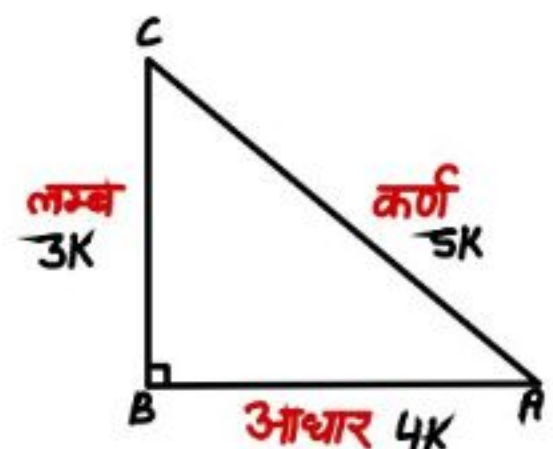
$$3 \cot A = 4$$

$$\cot A = \frac{4}{3} = \frac{\text{आधार}}{\text{लम्ब}}$$

$$\text{आधार} : \text{लम्ब} = 4 : 3$$

$$\text{आधार} = 4k$$

$$\text{लम्ब} = 3k$$



$\Delta ABC$  में,

पाइथागोरस प्रमेय से,

$$\Rightarrow AB^2 + BC^2 = AC^2$$

$$\Rightarrow (4K)^2 + (3K)^2 = AC^2$$

$$\Rightarrow 16K^2 + 9K^2 = AC^2$$

$$\Rightarrow 25K^2 = AC^2$$

$$\Rightarrow AC = \sqrt{25K^2}$$

$$\Rightarrow AC = 5K$$

$$\Rightarrow \sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{3K}{5K} = \frac{3}{5}$$

$$\Rightarrow \cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{4K}{5K} = \frac{4}{5}$$

$$\Rightarrow \tan A = \frac{\text{लम्ब}}{\text{आधार}} = \frac{3K}{4K} = \frac{3}{4}$$

$$\text{LHS} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{\left(\frac{16-9}{16}\right)}{\left(\frac{16+9}{16}\right)} = \frac{7}{25}$$

$$= \frac{7}{16} \times \frac{16}{25} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A$$

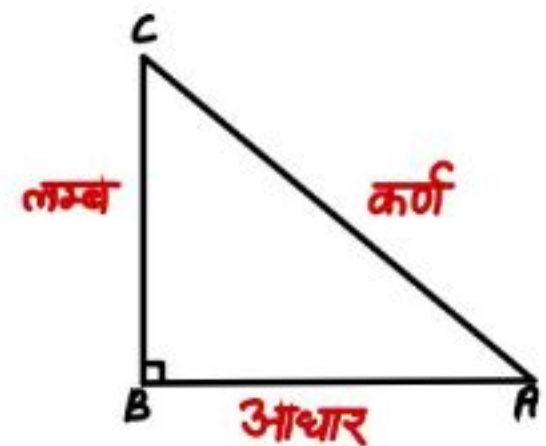
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{16-9}{25}$$

$$= \frac{7}{25}$$

$$\boxed{\text{LHS} = \text{RHS}}$$





प्रश्न-9 त्रिभुज ABC में जिसका कोण B समकोण है, यदि  $\tan A = \frac{1}{\sqrt{3}}$ , तो निम्नलिखित के मान ज्ञात कीजिए।

(i)  $\sin A \cos C + \cos A \sin C$

(ii)  $\cos A \cos C - \sin A \sin C$

हल:

$$\tan A = \frac{\text{लम्ब}}{\text{आधार}} = \frac{1}{\sqrt{3}}$$

$$\text{लम्ब} : \text{आधार} = 1 : \sqrt{3}$$

$$\text{लम्ब} = 1K$$

$$\text{आधार} = \sqrt{3}K$$

$\Delta ABC$  में,

पाइथागोरस प्रमेय से,

$$\Rightarrow AB^2 + BC^2 = AC^2$$

$$\Rightarrow (\sqrt{3}K)^2 + (1K)^2 = AC^2$$

$$\Rightarrow 3K^2 + 1K^2 = AC^2$$

$$\Rightarrow 4K^2 = AC^2$$

$$\Rightarrow AC = \sqrt{4K^2}$$

$$\Rightarrow \boxed{AC = 2K}$$

$$\Rightarrow \sin A = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{1K}{2K} = \frac{1}{2}$$

$$\Rightarrow \cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin C = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$$

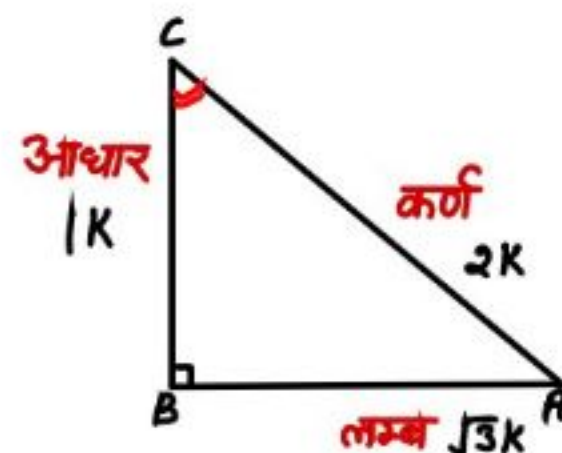
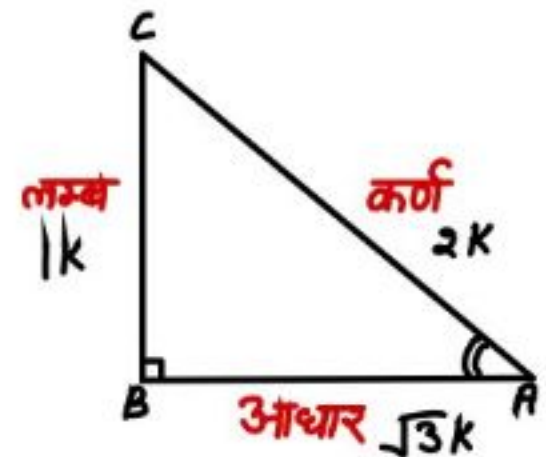
$$\Rightarrow \cos C = \frac{\text{आधार}}{\text{कर्ण}} = \frac{1K}{2K} = \frac{1}{2}$$

(i)  $\sin A \cos C + \cos A \sin C$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = \boxed{1}$$



$$(ii) \cos A \cos C - \sin A \sin C$$

$$\left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \boxed{0}$$

प्रश्न-10  $\triangle PQR$  में, जिसका कोण  $Q$  समकोण है,  $PR + QR = 25 \text{ cm}$  और  $PQ = 5 \text{ cm}$  है।  $\sin P$ ,  $\cos P$  और  $\tan P$  के मान ज्ञात कीजिए।

हल:-  $PR + QR = 25$

$$\Rightarrow \boxed{PR = 25 - QR} \quad - (1)$$

$\triangle PQR$  में,

पाइथागोरस प्रमेय से,

$$\begin{aligned} \Rightarrow PQ^2 + QR^2 &= PR^2 \\ \Rightarrow 5^2 + QR^2 &= (25 - QR)^2 \end{aligned}$$

$$\Rightarrow 25 + QR^2 = (25)^2 + (QR)^2 - 2(25)(QR)$$

$$\Rightarrow 25 = 625 - 50(QR)$$

$$\Rightarrow 50(QR) = 625 - 25$$

$$\Rightarrow 50(QR) = 600$$

$$\Rightarrow QR = \frac{600}{50}$$

$$\Rightarrow \boxed{QR = 12 \text{ cm}}$$

समीकरण (1) से  $\rightarrow$

$$PR = 25 - QR$$

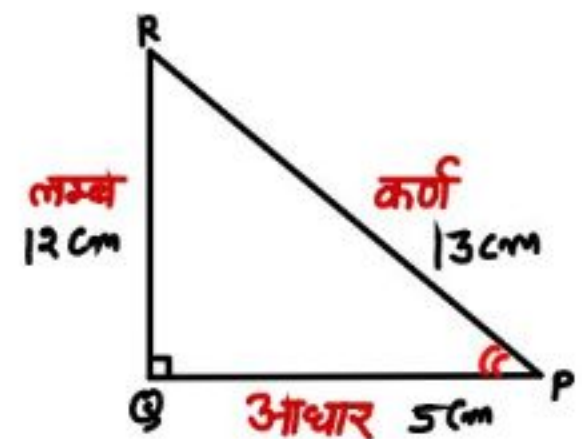
$$PR = 25 - 12$$

$$\boxed{PR = 13 \text{ cm}}$$

$$\Rightarrow \sin P = \frac{\text{लम्बा}}{\text{कर्ण}} = \frac{12}{13}$$

$$\Rightarrow \cos P = \frac{\text{आधार}}{\text{कर्ण}} = \frac{5}{13}$$

$$\Rightarrow \tan P = \frac{\text{लम्बा}}{\text{आधार}} = \frac{12}{5}$$



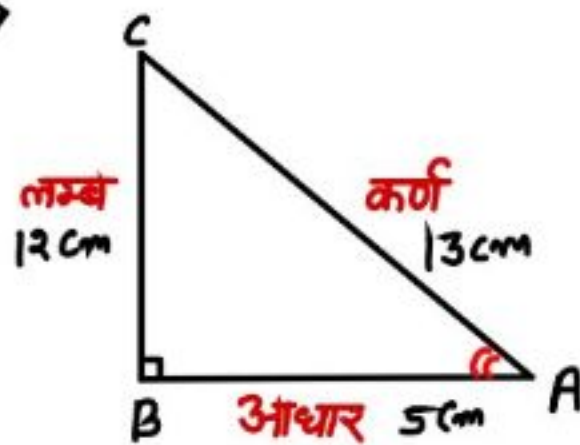
प्रश्न-11 बताइए कि निम्नलिखित कथन सत्य हैं या असत्य। कारण सहित अपने उत्तर की पुष्टि कीजिए।

(i)  $\tan A$  का मान सदैव 1 से कम होता है।  
असत्य

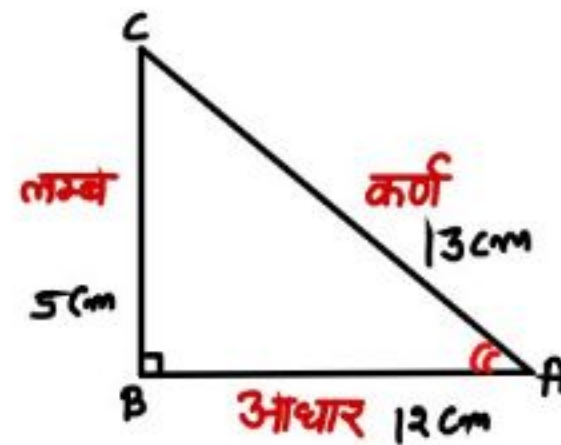


$\tan A$  का मान 1 से कम या अधिक हो सकता है।

उदाहरण →



$$\tan A = \frac{12}{5} \text{ जो कि 1 से अधिक है}$$



$$\tan A = \frac{5}{12} \Rightarrow \text{जो कि 1 से कम है।}$$

(ii) कोण A के किसी मान के लिए  $\sec A = \frac{13}{5}$   
सत्य

$$\sec A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{13}{5} \Rightarrow \text{कर्ण हमेशा आधार से बड़ा होता है।}$$

(iii)  $\cos A$ , कोण A के Cosecant के लिए प्रयुक्त एक संक्षिप्त रूप है।  
असत्य

$$\text{Cosine} \rightarrow \cos$$

$\sin$	$\rightarrow$	Sine
$\cos$	$\rightarrow$	Cosine
$\tan$	$\rightarrow$	Tangent
$\cot$	$\rightarrow$	Cotangent
$\sec$	$\rightarrow$	Secant
$\csc$	$\rightarrow$	Cosecant

(iv)  $\cot A$ ,  $\cot$  और A का गुणनफल है।  
असत्य

$\cot A$  में  $\cot$  और A का गुणनफल नहीं है।

$\cot$  और A एक साथ ही है।

(v) किसी भी कोण  $\theta$  के लिए  $\sin \theta = \frac{4}{3}$   
असत्य

$$\sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{4}{3}$$

$\Rightarrow$  कर्ण कभी भी लम्ब से छोटा नहीं हो सकता है।

**उदाहरण-6**  $\triangle ABC$  में जिसका कोण  $B$  समकोण है,  $AB = 5\text{cm}$  और  $\angle ACB = 30^\circ$ । भुजाओं  $BC$  और  $AC$  की लम्बाइयाँ ज्ञात कीजिए।

हल:-

$$\Rightarrow \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}}$$

$$\Rightarrow \sin 30^\circ = \frac{5}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{5}{AC}$$

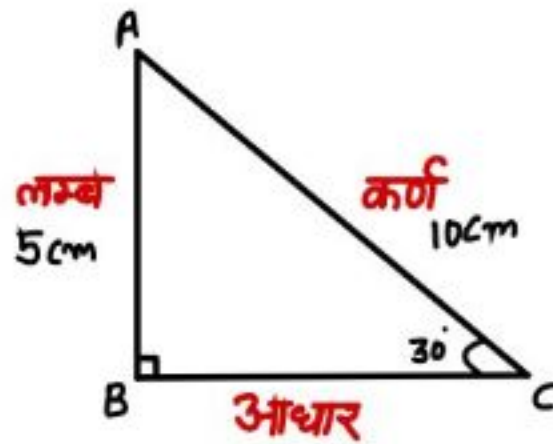
$$\Rightarrow AC = 5 \times 2 = 10\text{cm} \Rightarrow \boxed{AC = 10\text{cm}}$$

$$\Rightarrow \tan \theta = \frac{\text{लम्ब}}{\text{आधार}}$$

$$\Rightarrow \tan 30^\circ = \frac{5}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC}$$

$$\boxed{BC = 5\sqrt{3}\text{cm}}$$



	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

**उदाहरण-7**  $\triangle PQR$  में, जिसका कोण  $Q$  समकोण है,  $PQ = 3\text{cm}$  और  $PR = 6\text{cm}$  हैं।  $\angle QPR$  और  $\angle PRQ$  ज्ञात कीजिए।

हल:-

$$\Rightarrow \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}}$$

$$\Rightarrow \sin R = \frac{3}{6}$$

$$\Rightarrow \sin R = \frac{1}{2}$$

$$\Rightarrow \boxed{R = 30^\circ}$$

त्रिभुज के तीनों कोणों का योग  $= 180^\circ$

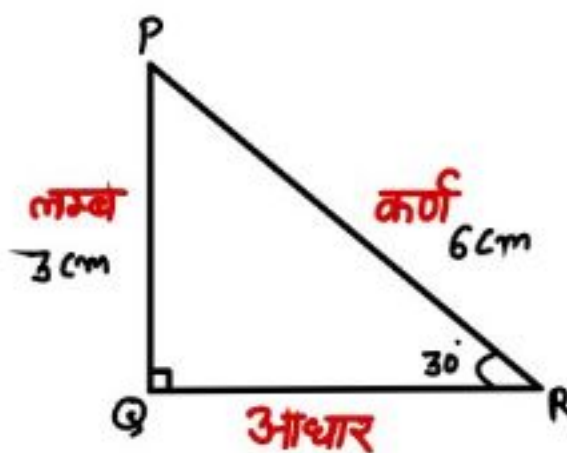
$$\Rightarrow \angle P + \angle Q + \angle R = 180^\circ$$

$$\Rightarrow \angle P + 90^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow \angle P + 120^\circ = 180^\circ$$

$$\angle P = 180^\circ - 120^\circ$$

$$\Rightarrow \boxed{\angle P = 60^\circ}$$



	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

**उदाहरण-8** यदि  $\sin(A-B) = \frac{1}{2}$ ,  $\cos(A+B) = \frac{1}{2}$ ;  $0^\circ < A+B \leq 90^\circ$ ,  $A > B$ , तो  $A$  और  $B$  ज्ञात कीजिए।

हल:-

$$\sin(A-B) = \frac{1}{2}$$

$$\cos(A+B) = \frac{1}{2}$$

$$\Rightarrow A-B = 30^\circ \text{ --- (1)}$$

$$\Rightarrow A+B = 60^\circ \text{ --- (2)}$$



समीकरण ①+② से →

$$\begin{aligned} A - B &= 30^\circ \\ A + B &= 60^\circ \\ \Rightarrow 2A &= 90^\circ \\ \Rightarrow A &= \frac{90}{2} = 45^\circ \end{aligned}$$

समीकरण ② से →

$$\begin{aligned} \Rightarrow 45^\circ + B &= 60^\circ \\ \Rightarrow B &= 60^\circ - 45^\circ \\ \Rightarrow B &= 15^\circ \end{aligned}$$

	0°	30	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

## प्रश्नावली 8.2

प्रश्न-1 निम्नलिखित के मान निकालिए :

(i)  $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$\begin{aligned} \Rightarrow \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ \\ \Rightarrow \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \\ \frac{3}{4} + \frac{1}{4} \\ \Rightarrow \frac{3+1}{4} = \frac{4}{4} = 1 \end{aligned}$$

(ii)  $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$\begin{aligned} \Rightarrow 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ \\ \Rightarrow 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ \Rightarrow 2 \times 1 = 2 \end{aligned}$$

(iii)  $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$\Rightarrow \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{1}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

	0°	30	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

$$\Rightarrow \frac{1}{\sqrt{}} \times \frac{\sqrt{3}}{2+\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{3}(1+\sqrt{3})}$$

$$\Rightarrow \frac{\sqrt{3}}{2\sqrt{3}(1+\sqrt{3})} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{4(\sqrt{3}+1)}$$

$$\Rightarrow \frac{\sqrt{6}}{4(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow \frac{\sqrt{6}(\sqrt{3}-1)}{4[(\sqrt{3})^2-1^2]}$$

$$\Rightarrow \frac{\sqrt{18} - \sqrt{6}}{4(3-1)}$$

$$\Rightarrow \boxed{\frac{3\sqrt{2} - \sqrt{6}}{8}}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

(iv)  $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\Rightarrow \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$\Rightarrow \frac{\left(\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}\right)}{\left(\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}\right)}$$

$$\Rightarrow \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} \Rightarrow \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} \times \frac{4 - 3\sqrt{3}}{4 - 3\sqrt{3}}$$

$$\Rightarrow \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$\Rightarrow \frac{(3\sqrt{3})^2 + (4)^2 - 2(3\sqrt{3})(4)}{9 \times 3 - 16}$$

$$\Rightarrow \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$$

$$\Rightarrow \boxed{\frac{43 - 24\sqrt{3}}{11}}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1



$$(V) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\Rightarrow \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\Rightarrow \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{(5 \times \frac{1}{4}) + 4\left(\frac{4}{3}\right) - 1}{\left(\frac{1}{4} + \frac{3}{4}\right)}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\left(\frac{1+3}{4}\right)}$$

$$\Rightarrow \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{4}{4}\right)} = \boxed{\frac{67}{12}}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

प्रश्न-२

$$(i) \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$(A) \sin 60^\circ \quad (B) \cos 60^\circ$$

$$(C) \tan 60^\circ \quad (D) \sin 30^\circ$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\frac{\frac{2}{\sqrt{3}}}{\frac{1}{1} + \frac{1}{3}}$$

$$\frac{\frac{2}{\sqrt{3}}}{\left(\frac{3+1}{3}\right)} = \frac{\frac{2}{\sqrt{3}}}{\left(\frac{4}{3}\right)} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{2\sqrt{3}}$$

$$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3} \sqrt{3}}{2\sqrt{3}} = \boxed{\frac{\sqrt{3}}{2}} = \boxed{\sin 60^\circ} \Rightarrow \boxed{A}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

(ii)  $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

- (A)  $\tan 90^\circ$  (C)  $\sin 45^\circ$   
(B) 1 (D) 0

$$\Rightarrow \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$$

$$= \frac{1 - (1)^2}{1 + (1)^2}$$

$$= \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0 \Rightarrow \boxed{D}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

(iii)  $\sin 2A = 2 \sin A$  तब सत्य होता है, जबकि A बराबर है:

- (A) 0° (B) 30°  
(C) 45° (D) 60°

हल:-

$$\Rightarrow \sin 0 = 2 \sin 0$$

$$\Rightarrow 0 = 2 \times 0$$

$$\Rightarrow 0 = 0 \rightarrow \boxed{A}$$

(iv)  $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$  बराबर है

- (A)  $\cos 60^\circ$  (B)  $\sin 60^\circ$   
(C)  $\tan 60^\circ$  (D)  $\sin 30^\circ$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$\Rightarrow \frac{2 \left( \frac{1}{\sqrt{3}} \right)}{1 - \left( \frac{1}{\sqrt{3}} \right)^2} = \frac{\left( \frac{2}{\sqrt{3}} \right)}{\left( 1 - \frac{1}{3} \right)}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$\Rightarrow \frac{3}{\sqrt{3}} = \frac{\sqrt{3} \sqrt{3}}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ \rightarrow \boxed{(C)}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

**प्रश्न-3** यदि  $\tan(A+B) = \sqrt{3}$  और  $\tan(A-B) = \frac{1}{\sqrt{3}}$ ,  $0^\circ < A+B \leq 90^\circ$ ,  $A > B$  तो A और B का मान ज्ञात कीजिए।

हल:-

$$\tan(A+B) = \sqrt{3} \quad \Rightarrow \tan(A-B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow A+B = 60^\circ - \textcircled{1} \quad \Rightarrow A-B = 30^\circ - \textcircled{2}$$

समीकरण ① व ② से  $\rightarrow$



$$\begin{aligned}
 A + B &= 60^\circ \\
 A - B &= 30^\circ \\
 \hline
 2A &= 90^\circ \\
 \Rightarrow A &= \frac{90}{2} = 45^\circ
 \end{aligned}$$

समीकरण (2) से  $\rightarrow$

$$\begin{aligned}
 \Rightarrow 45^\circ - B &= 30^\circ \\
 \Rightarrow 45^\circ - 30^\circ &= B \\
 \Rightarrow 15^\circ &= B \Rightarrow \boxed{B = 15^\circ}
 \end{aligned}$$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
cot	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
cosec	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

#### प्रश्न-4

(i)  $\sin(A+B) = \sin A + \sin B$

$A = 30^\circ, B = 60^\circ$

$\Rightarrow \sin 90^\circ = \sin 30^\circ + \sin 60^\circ$

$\Rightarrow 1 = \frac{1}{2} + \frac{\sqrt{3}}{2}$

$\Rightarrow 1 = \frac{1+\sqrt{3}}{2} \Rightarrow$  असत्य

(ii) 0 में वृद्धि होने के साथ  $\sin \theta$  के मान में भी वृद्धि होती है।

सत्य

(iii) 0 में वृद्धि होने के साथ  $\cos \theta$  के मान में भी वृद्धि होती है।

असत्य

(iv) 0 के सभी मानों पर  $\sin \theta = \cos \theta$

$\Rightarrow$  असत्य

$\sin \theta = \cos \theta$  केवल  $45^\circ$  पर बराबर होते हैं

अन्य पर बराबर नहीं होता है।

(v)  $A = 0^\circ$  पर  $\cot A$  परिभाषित नहीं है।

सत्य

$\cot 0^\circ = \infty$  (परिभाषित नहीं है)

उदाहरण-9  $\frac{\tan 65^\circ}{\cot 25^\circ}$  का मान ज्ञात कीजिए।

हल:-

$$\begin{aligned}
 &\frac{\tan 65^\circ}{\cot 25^\circ} \\
 &= \frac{\tan(90^\circ - 25^\circ)}{\cot 25^\circ}
 \end{aligned}$$

$$\begin{aligned}
 \sin(90^\circ - A) &= \cos A \\
 \cos(90^\circ - A) &= \sin A \\
 \tan(90^\circ - A) &= \cot A \\
 \cot(90^\circ - A) &= \tan A \\
 \sec(90^\circ - A) &= \csc A \\
 \csc(90^\circ - A) &= \sec A
 \end{aligned}$$



$$= \frac{\cot 25^\circ}{\cot 25^\circ} \quad [\because \tan(90^\circ - A) = \cot A]$$

$$= 1$$

**उदाहरण-10** यदि  $\sin 3A = \cos(A-26^\circ)$  हो जहाँ,  $3A$  एक न्यून कोण है तो  $A$  का मान ज्ञात कीजिए।

हल:-  $\Rightarrow \sin 3A = \cos(A-26^\circ)$   
 $\Rightarrow \cos(90^\circ - 3A) = \cos(A-26^\circ)$   
 $\Rightarrow 90^\circ - 3A = A - 26^\circ$   
 $\Rightarrow 90 + 26 = A + 3A$   
 $\Rightarrow 116 = 4A$   
 $\Rightarrow A = \frac{116}{4}$

$$A = 29^\circ$$

$$\begin{aligned} \sin(90^\circ - A) &= \cos A \\ \cos(90^\circ - A) &= \sin A \\ \tan(90^\circ - A) &= \cot A \\ \cot(90^\circ - A) &= \tan A \\ \sec(90^\circ - A) &= \operatorname{cosec} A \\ \operatorname{cosec}(90^\circ - A) &= \sec A \end{aligned}$$

**उदाहरण-11**  $\cot 85^\circ + \cos 75^\circ$  को  $0^\circ$  और  $45^\circ$  के बीच के कोणों के त्रिकोणमितीय अनुपातों के पदों में व्यक्त कीजिए।

हल:-

$$\begin{aligned} &\Rightarrow \cot 85^\circ + \cos 75^\circ \\ &\Rightarrow \cot(90^\circ - 5^\circ) + \cos(90^\circ - 15^\circ) \\ &\Rightarrow \tan 5^\circ + \sin 15^\circ \end{aligned}$$

### प्रश्नावली - 8.3

**प्रश्न-1**

(i)  $\frac{\sin 18^\circ}{\cos 72^\circ}$

$$\Rightarrow \frac{\sin 18^\circ}{\cos 72^\circ}$$

$$\Rightarrow \frac{\sin 18^\circ}{\cos(90^\circ - 18^\circ)}$$

$$\Rightarrow \frac{\sin 18^\circ}{\sin 18^\circ} = 1 \quad [\because \cos(90^\circ - A) = \sin A]$$

(ii)  $\frac{\tan 26^\circ}{\cot 64^\circ}$

$$\frac{\tan 26^\circ}{\cot 64^\circ}$$

$$= \frac{\tan 26^\circ}{\cot(90^\circ - 26^\circ)}$$

$$\Rightarrow \frac{\tan 26^\circ}{\tan 26^\circ} = \boxed{1} \quad [\because \cot(90^\circ - A) = \tan A]$$

$$(iii) \cos 48^\circ - \sin 42^\circ$$

$$\Rightarrow \cos 48^\circ - \sin 42^\circ$$

$$\Rightarrow \cos(90^\circ - 42^\circ) - \sin 42^\circ$$

$$\Rightarrow \sin 42^\circ - \sin 42^\circ \quad [\because \cos(90^\circ - A) = \sin A]$$

$$\Rightarrow \boxed{0}$$

$$(iv) \operatorname{cosec} 31^\circ - \sec 59^\circ$$

$$\Rightarrow \operatorname{cosec} 31^\circ - \sec 59^\circ$$

$$\Rightarrow \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$$

$$\Rightarrow \sec 59^\circ - \sec 59^\circ \quad [\because \operatorname{cosec}(90^\circ - A) = \sec A]$$

$$\Rightarrow \boxed{0}$$

प्रश्न-2 दिखाइए कि:-

$$(i) \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

$$LHS = \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$$

$$\Rightarrow \tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$$

$$[\tan(90^\circ - A) = \cot A]$$

$$\Rightarrow \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$\Rightarrow \cot 42^\circ \cot 67^\circ \frac{1}{\cot 42^\circ} \frac{1}{\cot 67^\circ}$$

$$[\tan A = \frac{1}{\cot A}]$$

$$\Rightarrow \boxed{1} = RHS$$

$$(ii) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$$

$$LHS = \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$\Rightarrow \cos(90^\circ - 52^\circ) \cos(90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$$

$$\Rightarrow \sin 38^\circ \sin 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$\Rightarrow \boxed{0} = RHS$$

प्रश्न-3 यदि  $\tan 2A = \cot(A - 18^\circ)$ , जहाँ  $2A$  एक न्यूनकोण है, तो  $A$  का माप ज्ञात कीजिए।

$$\underline{\text{हल}}:- \Rightarrow \tan 2A = \cot(A - 18^\circ)$$

$$\Rightarrow \cot(90^\circ - 2A) = \cot(A - 18^\circ) \quad [\because \tan A = \cot(90^\circ - A)]$$

$$\Rightarrow 90^\circ - 2A = A - 18^\circ$$

$$\Rightarrow 90^\circ + 18^\circ = A + 2A$$

$$\Rightarrow 108^\circ = 3A$$

$$\Rightarrow A = \frac{108^\circ}{3}$$

$$\Rightarrow \boxed{A = 36^\circ}$$



प्रश्न-4 यदि  $\tan A = \cot B$ , तो सिद्ध कीजिए  $A+B = 90^\circ$

हल:-

$$\Rightarrow \tan A = \cot B$$

$$\Rightarrow \cot(90^\circ - A) = \cot B$$

$$\Rightarrow 90^\circ - A = B$$

$$\Rightarrow \boxed{90^\circ = A+B}$$

प्रश्न-5 यदि  $\sec 4A = \operatorname{cosec}(A-20^\circ)$ , जहाँ  $4A$  एक न्यूनकोण है, तो  $A$  का मान ज्ञात करो।

हल:-

$$\Rightarrow \sec 4A = \operatorname{cosec}(A-20^\circ)$$

$$\Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A-20^\circ)$$

$$\Rightarrow 90^\circ - 4A = A - 20^\circ$$

$$\Rightarrow 90^\circ + 20^\circ = A + 4A$$

$$\Rightarrow 110^\circ = 5A$$

$$\Rightarrow A = \frac{110}{5}$$

$$\Rightarrow \boxed{A = 22^\circ}$$

प्रश्न-6 यदि  $A, B$  और  $C$  त्रिभुज  $ABC$  के अन्तः कोण हों, तो दिखाइए कि  $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$

हल:-

$$\text{कोणों का योग} = 180^\circ$$

$$A + B + C = 180^\circ$$

$$\boxed{B+C = 180^\circ - A} \quad \text{--- (1)}$$

$$\text{LHS} = \sin\left(\frac{B+C}{2}\right)$$

$$= \sin\left(\frac{180^\circ - A}{2}\right)$$

$$= \sin\left(\frac{180^\circ}{2} - \frac{A}{2}\right)$$

$$= \sin\left(90^\circ - \frac{A}{2}\right)$$

$$[\because \sin(90^\circ - A) = \cos A]$$

$$= \boxed{\cos\left(\frac{A}{2}\right)} = \text{RHS}$$

प्रश्न-7  $\sin 67^\circ + \cos 75^\circ$  को  $0^\circ$  और  $45^\circ$  के बीच के कोणों के त्रिकोणमितीय अनुपातों के पदों में व्यक्त कीजिए।

हल:-

$$\sin 67^\circ + \cos 75^\circ$$

$$= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$$

$$= \boxed{\cos 23^\circ + \sin 15^\circ}$$

$$[\sin(90^\circ - A) = \cos A]$$

$$[\cos(90^\circ - A) = \sin A]$$



**उदाहरण-12** अनुपातों  $\cos A$ ,  $\tan A$  और  $\sec A$  को  $\sin A$  के पदों में व्यक्त कीजिए।

हल :- (a)  $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \cos^2 A = 1 - \sin^2 A$$

$$\Rightarrow \boxed{\cos^2 A = \sqrt{1 - \sin^2 A}} \quad \text{--- ①}$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

(b)  $\Rightarrow \tan A = \frac{\sin A}{\cos A}$

$$\Rightarrow \boxed{\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}} \quad \text{--- ②} \quad \because [\cos^2 A = \sqrt{1 - \sin^2 A}]$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

(c)  $\sec A = \frac{1}{\cos A}$

$$\Rightarrow \boxed{\sec A = \frac{1}{\sqrt{1 - \sin^2 A}}} \quad \text{--- ③} \quad \because [\cos^2 A = \sqrt{1 - \sin^2 A}]$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

**उदाहरण-13** सिद्ध कीजिए कि  $\sec (1 - \sin A) (\sec A + \tan A) = 1$

हल :-

$$LHS = \sec (1 - \sin A) (\sec A + \tan A)$$

$$= \frac{1}{\cos A} \cdot (1 - \sin A) \left( \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)$$

$$= \frac{(1 - \sin A)}{\cos A} \cdot \left( \frac{1 + \sin A}{\cos A} \right)$$

$$= \frac{(1)^2 - (\sin A)^2}{(\cos A)^2}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$[\cos^2 A = 1 - \sin^2 A]$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1$$

**उदाहरण-14** सिद्ध कीजिए कि:

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$LHS = \frac{\cot A - \cos A}{\cot A + \cos A}$$

$$\begin{aligned}
 &= \frac{\left( \frac{\cos A}{\sin A} - \frac{\cos A}{1} \right)}{\left( \frac{\cos A}{\sin A} + \frac{\cos A}{1} \right)} \\
 &= \frac{\cos A \left( \frac{1}{\sin A} - 1 \right)}{\cos A \left( \frac{1}{\sin A} + 1 \right)} \\
 &= \frac{\left( \frac{1}{\sin A} - 1 \right)}{\left( \frac{1}{\sin A} + 1 \right)} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \sin^2 A + \cos^2 A &= 1 \\
 1 + \tan^2 A &= \sec^2 A \\
 1 + \cot^2 A &= \operatorname{cosec}^2 A
 \end{aligned}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

**उदाहरण-15** सर्वसमिका  $\sec^2 \theta = 1 + \tan^2 \theta$  का प्रयोग करके सिद्ध कीजिए कि:

$$\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$$

हल:-

$$\text{LHS} = \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1}$$

$$= \frac{\left( \frac{\sin \theta - \cos \theta + 1}{\cos \theta} \right)}{\left( \frac{\sin \theta + \cos \theta - 1}{\cos \theta} \right)}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\cos \theta} + \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta} - \frac{1}{\cos \theta}}$$

$$= \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta}$$

$$= \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1}$$

$$= \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + (\sec^2 \theta - \tan^2 \theta)}$$

$$\because [\sec^2 \theta = 1 + \tan^2 \theta]$$

$$\begin{aligned}
 \sin^2 A + \cos^2 A &= 1 \\
 1 + \tan^2 A &= \sec^2 A \\
 1 + \cot^2 A &= \operatorname{cosec}^2 A
 \end{aligned}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

$$= \frac{\tan \theta + \sec \theta - 1}{-(\sec \theta - \tan \theta) + (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}$$

$$= \frac{\tan \theta + \sec \theta - 1}{(\sec \theta - \tan \theta)(-1 + \sec \theta + \tan \theta)}$$

$$= \frac{\tan \theta + \sec \theta - 1}{(\sec \theta - \tan \theta)(-1 + \sec \theta + \tan \theta)}$$

$$= \frac{\tan \theta + \sec \theta - 1}{(\sec \theta - \tan \theta)(\tan \theta + \sec \theta - 1)}$$

$$= \frac{1}{\sec \theta - \tan \theta} = \text{RHS}$$



## प्रश्नावली 8.4

**प्रश्न-1** त्रिकोणमितीय अनुपातों  $\sin A$ ,  $\sec A$  और  $\tan A$  को  $\cot A$  के पदों में व्यक्त कीजिए।

हल:-

$$\tan A = \frac{1}{\cot A} \quad \text{--- (1)}$$

$$\Rightarrow 1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\Rightarrow \operatorname{cosec} A = \sqrt{1 + \cot^2 A} \quad \text{--- (2)}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\Rightarrow \boxed{\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}} \quad \text{--- (3) [समीकरण 3 से]}$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \left(\frac{1}{\cot A}\right)^2$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \sqrt{\frac{1 + \cot^2 A}{\cot^2 A}}$$

$$\Rightarrow \boxed{\sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}} \quad \text{--- (4)}$$

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ 1 + \tan^2 A &= \sec^2 A \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \end{aligned}$$

$$\begin{aligned} \tan A &= \frac{\sin A}{\cos A} \\ \cot A &= \frac{\cos A}{\sin A} \end{aligned}$$

$$\begin{aligned} \sin A &= \frac{1}{\operatorname{cosec} A} \\ \cos A &= \frac{1}{\sec A} \\ \tan A &= \frac{1}{\cot A} \end{aligned}$$

**प्रश्न-2**  $\angle A$  के अन्य सभी त्रिकोणमितीय अनुपातों को  $\sec A$  के पदों में लिखिए।

हल:-

$$\cos A = \frac{1}{\sec A} \quad \text{--- (1)}$$

$$\Rightarrow \sin^2 A + \cos^2 A = 1$$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \left(\frac{1}{\sec A}\right)^2$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\Rightarrow \sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \quad - (2)$$

$$1 + \tan^2 A = \sec^2 A$$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1} \quad - (3)$$

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}} \quad - (4)$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$\Rightarrow \operatorname{cosec} A = \frac{1}{\left( \frac{\sqrt{\sec^2 A - 1}}{\sec A} \right)}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}} \quad - (5)$$

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ 1 + \tan^2 A &= \sec^2 A \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \end{aligned}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

[ समीकरण (2) से ]

**प्रश्न-3** मान निकालिए!

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$\Rightarrow \frac{\sin^2 (90^\circ - 27^\circ) + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 (90^\circ - 17^\circ)}$$

$$\Rightarrow \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \sin^2 17^\circ} \quad \left[ \begin{array}{l} \sin (90^\circ - A) = \cos A \\ \cos (90^\circ - A) = \sin A \end{array} \right]$$

$$\Rightarrow \frac{1}{1} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= 1$$

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ 1 + \tan^2 A &= \sec^2 A \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \end{aligned}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$



$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$\Rightarrow \sin 25^\circ \cos(90^\circ - 25^\circ) + \cos 25^\circ \sin(90^\circ - 25^\circ)$$

$$\Rightarrow \sin 25^\circ \sin 25^\circ + \cos 25^\circ \cos 25^\circ$$

$$\Rightarrow \sin^2 25^\circ + \cos^2 25^\circ$$

$$\Rightarrow \boxed{1}$$

**प्रश्न-4** सही विकल्प चुनिए और अपने विकल्प की पुष्टि कीजिए।

(i)  $9 \sec^2 A - 9 \tan^2 A$  का मान है:

(A) 1 (B) 9 (C) 8 (D) 0

हल:-

$$= 9 \sec^2 A - 9 \tan^2 A$$

$$= 9(1 + \tan^2 A) - 9 \tan^2 A$$

$$= 9 + 9 \tan^2 A - 9 \tan^2 A$$

$$= \boxed{9}$$

(ii)  $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$  का मान है:

(A) 0 (B) 1 (C) 2 (D) -1

$$(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

$$= \frac{\cos \theta \sin \theta + \cos^2 \theta - \cos \theta + \sin^2 \theta + \sin \theta \cos \theta - \sin \theta + \sin \theta + \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{2 \sin \theta \cos \theta + 1 - 1}{\sin \theta \cos \theta}$$

$$= \boxed{2}$$

(iii)  $(\sec A + \tan A)(1 - \sin A)$  का मान है:

(A)  $\sec A$  (B)  $\sin A$  (C)  $\operatorname{cosec} A$  (D)  $\cos A$

हल:-

$$(\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) (1 - \sin A)$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$



$$= \left( \frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$= \frac{1^2 - \sin^2 A}{\cos A} \quad [\because (x+y)(x-y) = x^2 - y^2]$$

$$= \frac{1 - \sin^2 A}{\cos A} \quad [\cos^2 A = 1 - \sin^2 A]$$

$$= \frac{\cos^2 A}{\cos A} = \boxed{\cos A}$$

(iv)  $\frac{1 + \tan^2 A}{1 + \cot^2 A}$  सराबर है:

(A)  $\sec^2 A$  (B)  $-1$  (C)  $\cot^2 A$  (D)  $\tan^2 A$

हल :-

$$\frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \boxed{\tan^2 A}$$

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ 1 + \tan^2 A &= \sec^2 A \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \end{aligned}$$

$$\begin{aligned} \tan A &= \frac{\sin A}{\cos A} \\ \cot A &= \frac{\cos A}{\sin A} \end{aligned}$$

$$\begin{aligned} \sin A &= \frac{1}{\operatorname{cosec} A} \\ \cos A &= \frac{1}{\sec A} \\ \tan A &= \frac{1}{\cot A} \end{aligned}$$

**प्रश्न-5**

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\text{LHS} = (\operatorname{cosec} \theta - \cot \theta)^2$$

$$= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)$$

$$= \left( \frac{1 - \cos \theta}{\sin \theta} \right)^2$$

$$\begin{aligned}
&= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\
&= \frac{(1 - \cos \theta)^2}{(1^2 - \cos^2 \theta)} \\
&= \frac{(1 - \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)} \quad [\because (x+y)(x-y) = x^2 - y^2] \\
&= \frac{(1 - \cos \theta)}{(1 + \cos \theta)} = \text{RHS}
\end{aligned}$$

$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\text{LHS} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A}$$

$$= \frac{\cos^2 A + 1^2 + \sin^2 A + 2(1) \sin A}{(1 + \sin A) \cos A}$$

$$= \frac{1 + 1 + 2 \sin A}{(1 + \sin A) \cos A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{2 + 2 \sin A}{(1 + \sin A) \cos A}$$

$$= \frac{2(1 + \sin A)}{(1 + \sin A) \cos A}$$

$$= 2 \left( \frac{1}{\cos A} \right)$$

$$= \boxed{2 \sec A} = \text{RHS}$$

$$\begin{aligned}
\sin^2 A + \cos^2 A &= 1 \\
1 + \tan^2 A &= \sec^2 A \\
1 + \cot^2 A &= \operatorname{cosec}^2 A
\end{aligned}$$

$$\begin{aligned}
\tan A &= \frac{\sin A}{\cos A} \\
\cot A &= \frac{\cos A}{\sin A}
\end{aligned}$$

$$\begin{aligned}
\sin A &= \frac{1}{\operatorname{cosec} A} \\
\cos A &= \frac{1}{\sec A} \\
\tan A &= \frac{1}{\cot A}
\end{aligned}$$



$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\text{LHS} = \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$$

$$= \frac{\left( \frac{\sin \theta}{\cos \theta} \right)}{\left( 1 - \frac{\cos \theta}{\sin \theta} \right)} + \frac{\left( \frac{\cos \theta}{\sin \theta} \right)}{\left( 1 - \frac{\sin \theta}{\cos \theta} \right)}$$

$$= \frac{\left( \frac{\sin \theta}{\cos \theta} \right)}{\left( \frac{\sin \theta - \cos \theta}{\sin \theta} \right)} + \frac{\left( \frac{\cos \theta}{\sin \theta} \right)}{\left( \frac{\cos \theta - \sin \theta}{\cos \theta} \right)}$$

$$= \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta - \cos \theta} + \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{\cos \theta - \sin \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}$$

$$= \frac{\sin^3 \theta - \cos^3 \theta}{\cos \theta \sin \theta (\sin \theta - \cos \theta)}$$

$$[x^3 - y^3 = (x - y)(x^2 + y^2 + xy)]$$

$$= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\cos \theta \sin \theta (\sin \theta - \cos \theta)}$$

$$= \frac{1 + \sin \theta \cos \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta} + \frac{\sin \theta \cos \theta}{\cos \theta \sin \theta}$$

$$= \boxed{\sec \theta \operatorname{cosec} \theta + 1} = \text{RHS}$$

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ 1 + \tan^2 A &= \sec^2 A \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \end{aligned}$$

$$\begin{aligned} \tan A &= \frac{\sin A}{\cos A} \\ \cot A &= \frac{\cos A}{\sin A} \end{aligned}$$

$$\begin{aligned} \sin A &= \frac{1}{\operatorname{cosec} A} \\ \cos A &= \frac{1}{\sec A} \\ \tan A &= \frac{1}{\cot A} \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{1 + \sec A}{\sec A} &= \frac{\sin^2 A}{1 - \cos A} \\
 &\text{RHS} \\
 &= \frac{\sin^2 A}{1 - \cos A} \\
 &= \frac{1^2 - \cos^2 A}{1 - \cos A} \\
 &\quad [\because x^2 - y^2 = (x+y)(x-y)] \\
 &= \frac{(1 + \cos A)(1 - \cos A)}{(1 - \cos A)} \\
 &= \boxed{1 + \cos A} \\
 &= \frac{1}{1} + \frac{1}{\sec A} \\
 &= \frac{\sec A + 1}{\sec A} = \text{LHS}
 \end{aligned}$$

2<sup>nd</sup> Method

$$\begin{aligned}
 \text{LHS} &= \frac{1 + \sec A}{\sec A} \\
 &= \frac{1}{\sec A} + \frac{\sec A}{\sec A} \\
 &= \boxed{\cos A + 1}
 \end{aligned}$$

$$\text{RHS} = \text{LHS}$$

$$\text{(v)} \quad \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

$$\begin{aligned}
 \text{LHS} &= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} \\
 &= \frac{\left( \frac{\cos A - \sin A + 1}{\sin A} \right)}{\left( \frac{\cos A + \sin A - 1}{\sin A} \right)} \\
 &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} \\
 &= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}
 \end{aligned}$$

$$\begin{aligned}
 \sin^2 A + \cos^2 A &= 1 \\
 1 + \tan^2 A &= \sec^2 A \\
 1 + \cot^2 A &= \operatorname{cosec}^2 A
 \end{aligned}$$

$$\begin{aligned}
 \tan A &= \frac{\sin A}{\cos A} \\
 \cot A &= \frac{\cos A}{\sin A}
 \end{aligned}$$

$$\begin{aligned}
 \sin A &= \frac{1}{\operatorname{cosec} A} \\
 \cos A &= \frac{1}{\sec A} \\
 \tan A &= \frac{1}{\cot A}
 \end{aligned}$$



$$\begin{aligned}
&= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1} \quad [\because 1 + \cot^2 A = \operatorname{cosec}^2 A] \\
&= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{(\cot A - \operatorname{cosec} A + 1)} \\
&= \frac{(\cot A + \operatorname{cosec} A) [1 - (\operatorname{cosec} A - \cot A)]}{(\cot A - \operatorname{cosec} A + 1)} \\
&= \frac{(\cot A + \operatorname{cosec} A) (1 - \operatorname{cosec} A + \cot A)}{(\cot A - \operatorname{cosec} A + 1)} \\
&= \frac{(\cot A + \operatorname{cosec} A) (\cot A - \operatorname{cosec} A + 1)}{(\cot A - \operatorname{cosec} A + 1)} \\
&= \boxed{(\cot A + \operatorname{cosec} A)} = \text{RHS}
\end{aligned}$$

$$(vi) \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \sec \theta + \tan \theta$$

$$\begin{aligned}
\text{LHS} &= \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} \times \frac{1 + \sin \theta}{1 + \sin \theta} \\
&= \sqrt{\frac{(1 + \sin \theta)^2}{1^2 - \sin^2 \theta}} \\
&= \frac{1 + \sin \theta}{\sqrt{1 - \sin^2 \theta}} \\
&= \frac{1 + \sin \theta}{\sqrt{\cos^2 \theta}} \\
&= \frac{1 + \sin \theta}{\cos \theta} \\
&= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\
&= \boxed{\sec \theta + \tan \theta} = \text{RHS}
\end{aligned}$$

$$\begin{aligned}
\sin^2 \theta + \cos^2 \theta &= 1 \\
1 + \tan^2 \theta &= \sec^2 \theta \\
1 + \cot^2 \theta &= \operatorname{cosec}^2 \theta
\end{aligned}$$

$$\begin{aligned}
\tan \theta &= \frac{\sin \theta}{\cos \theta} \\
\cot \theta &= \frac{\cos \theta}{\sin \theta}
\end{aligned}$$

$$\begin{aligned}
\sin \theta &= \frac{1}{\operatorname{cosec} \theta} \\
\cos \theta &= \frac{1}{\sec \theta} \\
\tan \theta &= \frac{1}{\cot \theta}
\end{aligned}$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$LHS = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\tan [1 - 2(1 - \cos^2 \theta)]}{(2 \cos^2 \theta - 1)}$$

$$= \frac{\tan \theta (1 - 2 + 2 \cos^2 \theta)}{2 \cos^2 \theta - 1}$$

$$= \frac{\tan \theta (2 \cos^2 \theta - 1)}{(2 \cos^2 \theta - 1)}$$

$$= \boxed{\tan \theta} = RHS$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$LHS = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$= 1 + 1 + \cot^2 A + 2 \cdot \frac{1}{\operatorname{cosec} A} \operatorname{cosec} A + 1 + \tan^2 A + 2 \cdot \frac{1}{\sec A} \sec A$$

$$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$$

$$= 7 + \cot^2 A + \tan^2 A$$

$$= \boxed{7 + \tan^2 A + \cot^2 A} = RHS$$

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{1}{\cot A}$$

$$(ix) (\operatorname{Cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$\text{LHS} = (\operatorname{Cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left( \frac{1}{\sin A} - \frac{\sin A}{1} \right) \left( \frac{1}{\cos A} - \frac{\cos A}{1} \right)$$

$$= \left( \frac{1 - \sin^2 A}{\sin A} \right) \left( \frac{1 - \cos^2 A}{\cos A} \right)$$

$$[\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$$= \boxed{\cos A \times \sin A}$$

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\left( \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right)}$$

$$= 1 \times \left( \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \right)$$

$$= \frac{\sin A \times \cos A}{1}$$

$$= \boxed{\sin A \times \cos A}$$

$$\text{LHS} = \text{RHS}$$

$$(x) \left( \frac{1 + \tan^2 A}{1 + \cot^2 A} \right)^2 = \left( \frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

सल :-

$$\frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{Cosec}^2 A}$$

$$= \frac{\left( \frac{1}{\cos^2 A} \right)}{\left( \frac{1}{\sin^2 A} \right)}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \boxed{\tan^2 A}$$



$$\left( \frac{1 - \tan A}{1 - \cot A} \right)^2$$

$$= \left( \frac{1 - \frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} \right)^2$$

$$= \left( \frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} \right)^2$$

$$= \left( \frac{\cos A - \sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} \right)^2$$

$$= \left[ -\frac{(\sin A - \cos A)}{\cos A} \times \frac{\sin A}{\sin A - \cos A} \right]^2$$

$$= \left( -\frac{\sin A}{\cos A} \right)^2 = (-\tan A)^2$$

$$= \boxed{\tan^2 A}$$